

Astrometric asteroid masses: a simultaneous determination

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ABSTRACT

Using over 89 million astrometric observations of 349 737 numbered minor planets, an attempt was made to determine the masses of 230 of them by simultaneously solving for corrections to all orbital elements and the masses. For 132 minor planets an acceptable result was obtained, 50 of which appear to be new.

Key words. astrometry - celestial mechanics - minor planets, asteroids: general

1. Introduction

The astrometric determination of asteroid masses is the technique of calculating the mass of asteroids (hereafter called "perturbing asteroids" or "perturbers") from the gravitational perturbations they exert on the motion of other asteroids (called "test asteroids").

The usual procedure to calculate the mass of one asteroid is, first, to identify potential test asteroids by searching for close approaches with the perturbing asteroids and then computing some parameter (usually the deflection angle or the change in mean motion) characterising the effect of the perturbing asteroid, see e.g. Galád and Gray (2002). Promising cases are then further investigated to see if they yield meaningful results, since the outcome also depends on the number, distribution and quality of the available astrometric observations of the test asteroid. Asteroids not making close approaches but moving in near-commensurable orbits can also be useful for mass determinations (Kochetova 2004).

A better but more laborious way to find suitable close encounters is to integrate the orbits of test asteroids over the period of available observations with and without the perturbations of the perturbing asteroid (Michalak 2000). Significant differences in the geocentric coordinates between the two calculations indicate cases that should be investigated further.

Initially, asteroid masses were computed from their perturbations on one other asteroid, the first instance being the mass of 4 Vesta computed from the orbit of 197 Arete by Hertz (1968). When several mass values for the same asteroid are calculated from a number of test asteroids, the usual practice is to compute a weighted mean along with the associated uncertainty as the final value, as in Michalak (2000). Mass determinations where one or a few masses are determined simultaneously from a number of test asteroids have been attempted on a modest scale in the past—see Sitarski and Todorovic-Juchniewicz (1995), Goffin (2001) and others.

This paper gives the results of an attempt to simultaneously determine a large number of asteroid masses (230) from a substantial number of test asteroids (349 737). Sect. 2 explains the basic principles and problems of simultaneous asteroid mass determination and in Sect. 3 the details of the actual calculations

are given. Sect. 4 briefly deals with the observations used and explains the statistical analysis of the residuals that was used to assign weights to, or reject observations. In Sect. 5 the results are presented and Sect. 6 discusses them.

Throughout this paper, asteroid masses are expressed in units of 10^{-10} solar masses (M_{\odot}) and generally given to 3 decimal places.

2. Simultaneous mass determination

The principle of the simultaneous determination of asteroid masses is simple: use the astrometric observations of N test asteroids to determine amongst others the masses of M perturbing asteroids. This direct approach has a number of distinct advantages. First, there is no need to search for close encounters to the M perturbers, assess their efficiency and select promising pairs for the actual calculations (the same can be said for near-commensurable orbits). All test asteroids are treated equal and close approaches will appear anyway when numerically integrating the N orbits. Next, one doesn't have to compute weighted means of masses and their standard deviations. Also, the mutual correlations between the M masses are taken into account in the global solution. But all this comes at a cost: computer resources!

2.1. The normal equations

In the process of mass determination, the equations of condition of the linearised problem are usually transformed into the normal equations :

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b} \quad (1)$$

The coefficient matrix \mathbf{A} is square, positive definite and symmetric about the main diagonal. For an "ordinary" mass determination ($N = 1, M = 1$) it is of size 7×7 (rows \times columns). Note that the coefficients below the main diagonal do not need to be stored, but that part of the matrix is often used as temporary storage when solving the equations or computing the inverse \mathbf{A}^{-1} .

The normal equations for simultaneous mass determination ($N > 1, M \geq 1$) are easily assembled from the normal equations of the individual test asteroids. Sitarski and Todorovic-Juchniewicz (1992) give a schematic representation for a simple

case ($N = 2, M = 1$). In general, \mathbf{A} is of size $(6N+M) \times (6N+M)$. While this is not prohibitive for modest applications, it is clear that \mathbf{A} will grow rapidly in size with large values of M and especially N , and eventually exceed the available computer memory.

The full normal equations (1) can be written as:

$$\begin{pmatrix} A_{1,1} & 0 & 0 & \cdots & 0 & A_{1,M} \\ 0 & A_{2,2} & 0 & \cdots & 0 & A_{2,M} \\ 0 & 0 & A_{3,3} & \cdots & 0 & A_{3,M} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A_{N,N} & A_{N,M} \\ A_{1,M}^T & A_{2,M}^T & A_{3,M}^T & \cdots & A_{N,M}^T & A_{M,M} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_N \\ X_M \end{pmatrix} = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ \vdots \\ B_N \\ B_M \end{pmatrix} \quad (2)$$

where:

- $A_{1,1}, \dots, A_{N,N}$ are the 6×6 blocks of the partial derivatives for the 6 orbital elements of the N test asteroids
- $A_{1,M}, \dots, A_{N,M}$ (each of size $6 \times M$) and $A_{M,M}$ (size $M \times M$) correspond to the M masses
- X_1, \dots, X_N are the variables pertaining to the N asteroids, namely the corrections to the 6 orbital elements (or components of the state vector) at epoch
- X_M are the variables pertaining to all perturbing asteroids, namely the corrections to the M masses.

The zeros represent of course 6×6 blocks of zeros. The matrix clearly contains a staggering amount of elements filled with zeros!

2.2. Solving the normal equations

One way to reduce computer storage is to store only the non-zero blocks of \mathbf{A} :

$$\mathbf{A} = \begin{pmatrix} A_{1,1} & A_{1,M} \\ A_{2,2} & A_{2,M} \\ A_{3,3} & A_{3,M} \\ \vdots & \vdots \\ A_{N,N} & A_{N,M} \\ & A_{M,M} \end{pmatrix} \quad (3)$$

When stored as a single array, the size of \mathbf{A} is thus reduced to $(6N + M) \times (6 + M)$, or, if $A_{M,M}$ is stored separately, to $6N(M + 6) + M \times M$. The Cholesky decomposition algorithm used to invert matrix \mathbf{A} was adapted to cope with this new form. In that way, long test calculations were done up to $N = 300\,000$, $M = 105$ (this proved to be a practical upper limit). Then, following a suggestion by E. Myles Standish, a formulation was adopted that only requires a matrix of dimensions $(6 + M) \times (6 + M)$!

The normal equations (2) can be written as the pair:

$$\begin{cases} A_{i,i}X_i + A_{i,M}X_M = B_i, & i = 1, N \\ \sum_i (A_{i,M}^T X_i) + A_{M,M}X_M = B_M \end{cases} \quad (4)$$

The first step is to eliminate the X_i 's from the second equation. From the first, we get:

$$X_i = A_{i,i}^{-1}(B_i - A_{i,M}X_M) \quad (5)$$

Substituting this in the second and rearranging gives:

$$X_M = \left[A_{M,M} - \sum_i (A_{i,M}^T A_{i,i}^{-1} A_{i,M}) \right]^{-1} \left[B_M - \sum_i (A_{i,M}^T A_{i,i}^{-1} B_i) \right] \quad (6)$$

With the X_M 's one can now solve eq. (5) for the X_i 's.

2.3. Standard deviations and correlations

The formal standard deviations and correlations come from the inverse of the \mathbf{A} -matrix. Representing the components of \mathbf{A}^{-1} by α 's, we have from the definition:

$$\begin{pmatrix} A_{i,i} & A_{i,M} \\ A_{i,M}^T & A_{M,M} \end{pmatrix} \begin{pmatrix} \alpha_{i,i} & \alpha_{i,M} \\ \alpha_{i,M}^T & \alpha_{M,M} \end{pmatrix} = I \quad (7)$$

The multiplication gives:

$$\begin{aligned} A_{i,i}\alpha_{i,i} + A_{i,M}\alpha_{i,M}^T &= I \\ A_{i,i}\alpha_{i,M} + A_{i,M}\alpha_{M,M} &= 0 \\ A_{i,M}^T\alpha_{i,i} + A_{M,M}\alpha_{i,M}^T &= 0 \\ A_{i,M}^T\alpha_{i,M} + A_{M,M}\alpha_{M,M} &= I \end{aligned} \quad (8)$$

The second equation of (8) gives:

$$\alpha_{i,M} = -A_{i,i}^{-1}A_{i,M}\alpha_{M,M} \quad (9)$$

Substituting this in the fourth yields:

$$\alpha_{M,M} = \left[A_{M,M} - \sum_i (A_{i,M}^T A_{i,i}^{-1} A_{i,M}) \right]^{-1} \quad (10)$$

which has already been calculated and inverted in (6).

With (9) one gets $\alpha_{i,i}$ from the first equation of (8):

$$\alpha_{i,i} = -A_{i,i}^{-1}(I - A_{i,M}\alpha_{i,M}^T) \quad (11)$$

The sequence of the computation is thus: (10), (9), (11).

2.4. Test asteroids

The test asteroids are those among the first 350 000 numbered asteroids with perihelion distance smaller than 6.0 au (to exclude distant objects such as KBO's ...), a total of 349 737. Their orbits were improved using perturbations by the major planets, Ceres, Pallas and Vesta, before using their orbital elements and observations as input for the simultaneous mass determination.

The positions of the eight major planets and the Moon are taken from the JPL ephemeris DE422, which is an extended version of DE421 (Folkner et al. 2008). The coordinates of the three largest asteroids initially come from a separate orbit improvement of these bodies.

2.5. Perturbing asteroids

Selecting the perturbing asteroids is basically quite simple: just take those with the largest mass. Reliable masses have been calculated for a number of asteroids, but for most others one must rely on estimates based on the diameter and the taxonomic class. Mass estimates for these asteroids were computed assuming a bulk density of 1.80 g/cm³ for class C (including B, D, F, G, P, T and X), 2.20 g/cm³ for class S (including A, E, K, Q, R and V) and 4.20 g/cm³ for class M.

Diameters were selected from the sources given in Baer et al. (2011), or from occultation results (Broughton 2011), or else from the results of the WISE (Masiero et al. 2011) or IRAS (Tedesco et al. 1992) surveys. For 4 Vesta the value from the Dawn spacecraft was used (Russell et al. 2012). The diameter of 52 Europa is from Merline et al. (2013) and that of 349 Dembowska from Majaess et al. (2008). In case the dimensions of a tri-axial ellipsoid were given, the equivalent diameter (of a sphere of the same volume) was calculated. When none of these

sources gave a diameter, a nominal value D was computed from the absolute magnitude H :

$$\log D = 3.62 - 0.20H$$

In this way, mass estimates were calculated for the first 100 000 numbered minor planets and those with mass $> 0.005 \times 10^{-10} M_{\odot}$ and perihelion distance $q < 6.0$ au, a total of 257, were withheld as perturbers for a first test run with the 349 737 test asteroids. After a few iterations it turned out that 279 Thule and the Trojans in the list didn't yield acceptable masses, so a new list was created with $q < 4.0$ and these 230 asteroids were used in the final calculations.

Though it would be possible to include the masses of the major planets in the solution, they were held fixed at the DE422 values.

3. Computations

The basics of the computations have been given in many previous papers, amongst others in Goffin (1991) and Goffin (2001). The force model includes the Newtonian attraction due to the Sun, the 8 major planets and the Moon, and the M perturbing asteroids (one less when the test asteroid is one of the perturbers). The heliocentric coordinates of the major planets and the Moon are stored in a planetary file together with those of the M perturbing asteroids. Also taken into account are the first-order relativistic effects due to the Sun as given in Ivantsov (2008) and the effects of the solar oblateness.

The longest part of the calculations is the numerical integration of the N test asteroid orbits and of the differential equations of all the required partial derivatives. These calculations only became feasible with the advent of personal computers with multi-core processors.

By the very nature of the problem, the solution is obtained by successive approximations. Each iteration cycle consists of four consecutive steps, each actually corresponding to a computer programme:

- $P1$: Numerical integration of the N test asteroids
- $P2$: Solution of the normal equations
- $P3$: Recomputation of the coordinates of the perturbing asteroids
- $P4$: Statistical analysis of the residuals.

The programme $P1$ takes care of the numerical integration of the N orbits. The mutual distances between perturbing and test asteroids are monitored and the step size is decreased when necessary. Orbital elements of all asteroids and masses of the perturbing asteroids are read in from the elements file on disk. The observations are weighted using the results of the statistical analysis, after correcting for any magnitude effects and systematic bias (see Sect. 4). The normal equations for each test asteroid are output to a disk file and all residuals ($O - C$) are stored in the residuals file together with relevant information (observatory code, time, visual magnitude of the asteroid, etc.).

Programme $P2$ reads in the separate normal equations, assembles the normal equations of the global solution, inverts the coefficient matrices and computes the $6N + M$ unknowns (initially 2 098 652 in total), their standard deviations and the correlation coefficients. This is actually done twice during each iteration. After the first solution, the validity of the resulting M masses is checked. Following Baer and Chesley (2008), a mass is considered acceptable when its *significance*, this being the ratio of the mass to its standard deviation, is larger than 2

($s_i = m_i/\sigma_{m_i} > +2.0$), and if the resulting density lies between 0.5 and 8.0 g/cm³. The rows and columns of the normal matrix corresponding to unacceptable masses are then modified (by filling in 0's or 1), and the right-hand side is changed so that the second solution will force them back to their initial (estimated) values. The improved masses and orbital elements from this solution are stored in the elements file.

The third programme ($P3$) uses the improved elements of the M perturbing asteroids to compute new heliocentric rectangular coordinates and copies them into the planetary file.

Programme $P4$ uses the residuals file to perform a statistical analysis of the performance of all observatories; the detailed description is the subject of Sect. 4. The results are stored on disk and used by $P1$ to assign weights to the observations. This analysis is done every 3 iterations. The sequence of the computations is thus: $3 \times (P1 + P2 + P3) + P4$.

4. Observations, weights and outlier rejection

4.1. Astrometric observations

The principal source of minor-planet observations are the files maintained at the Minor Planet Center (Cambridge, Mass.); the version of January 2013 was used. Throughout the years I have re-reduced most of the older observations of the first 1000 numbered asteroids made since their discovery and published in the open literature. They consist of series of visual micrometer and meridian observations, and a few series of photographic observations. Wherever possible, they were re-reduced with the aid of modern star catalogues. No further details will be given here. These observations were added to – and replaced some of – the data from the MPC. For the optical observations, those given to a precision worse than 0.01s in right ascension or 0.1'' in declination were not used. The total number of observations of the N test asteroids is over 89.4 million.

In the past, most researchers have used various methods for rejecting outliers and assigning weights to the observations. Some have used an iterative $n\sigma$ rejection criterion, or have assigned a-priori weights depending on the type of observation and/or the time period. Such simplifications may be easy to implement but do not reflect the real accuracy of the observations. Relevant weights must therefore be derived from a statistical analysis of the residuals resulting from orbit improvements. Since these two steps are interdependent, this is an iterative procedure.

Serious thoughts to the subject were first given by M. Carpino¹ and a thorough analysis was presented in Carpino et al. (2003). However, at that time I decided to continue my own statistical analysis for two reasons. The Carpino et al. paper was not yet published when I embarked on the project of mass determinations, so no practical weighting scheme was available. Also, the many old observations which I had re-reduced are mostly not present in the MPC data set and thus I had to provide their statistics myself.

4.2. Grouping of observations

The complete set of observations must be divided into groups with (supposedly) homogeneous properties, i.e. by instrument (observatory, telescope), observation and reduction method, and further by any type of information that can be relevant (observer, type of measuring instrument, number of readings taken etc.).

¹ Then on his web site; not available any more now.

In practice, all this information is not readily available and has not been stored with the current observation format of the Minor Planet Center. Therefore a division was made per observatory code (which can thus refer to different instruments and observers) and further by the following 4 observational methods: transit and meridian (T), visual micrometer (M), photographic (P) and CCD (C).

Each of these observation groups was further split up into a number of time bins as follows. A first time bin was "filled" until its time length exceeded 2500 days *or* the total number of observations exceeded 100 000; then the next bin was begun. The first condition limits the time length of the bins for observatories with few observations, while the second allows for a more detailed analysis of the more productive surveys. Time bins with less than 50 observations were considered too small for analysis and were combined into the 4 groups of observatory code 500 Geocentric. This code groups all observations that have been corrected for parallax and the reference to the original observatory is thus lost anyway.

4.3. Statistical analysis

One analysis cycle consists of the following steps.

- Using the residuals file created by *PI*, the observations are grouped in time bins as described above.
- Each time bin of each group is inspected for outliers. The basic idea is that each bin consists of a population of "good" data having Gaussian distribution and a population of outliers. To remove these outliers, the kurtosis parameter k is used (see Carpino et al. (2003) for a definition). Its value is equal to 3 for a normal distribution, and substantially larger when outliers are present. The calculation of k and the outlier removal is done iteratively. The residual with the largest absolute deviation from the mean μ is removed and μ and k are recomputed. This is repeated until $k \leq 3$ or, as a safety measure, until k starts to increase rather than decrease.
- The complete set of residuals of each group (observatory code + observation technique) is then analysed for a (possible) magnitude effect. This is an observational bias whereby a measurement error is correlated with the object's brightness. Such an effect is to be expected for observations where the right ascension is measured by timing transits, such as meridian and most micrometer observations. It can also occur in photographic and CCD astrometry due to imperfections in the optical system, e.g. coma. To model the magnitude effect, a least-squares regression line of the form

$$R = a + bM_V$$

is computed for both right ascension and declination residuals (R is thus $\cos \delta \Delta \alpha$ or $\Delta \delta$). The quantity M_V is the computed visual magnitude of the minor planet. In addition are computed: the total magnitude range ΔM of the set, the standard deviation σ_b of b , the correlation coefficient r (Pearson) and the statistic

$$t = r/\sigma_r = r\sqrt{(N-2)/(1-r^2)}$$

where N is the number of residuals. A magnitude effect is accepted as significant if:

$$\Delta M > 4.0, |t| > 2, \text{ and } |b|/\sigma_b > 2$$

Fig. 1 gives an example of a substantial magnitude effect in right ascension.

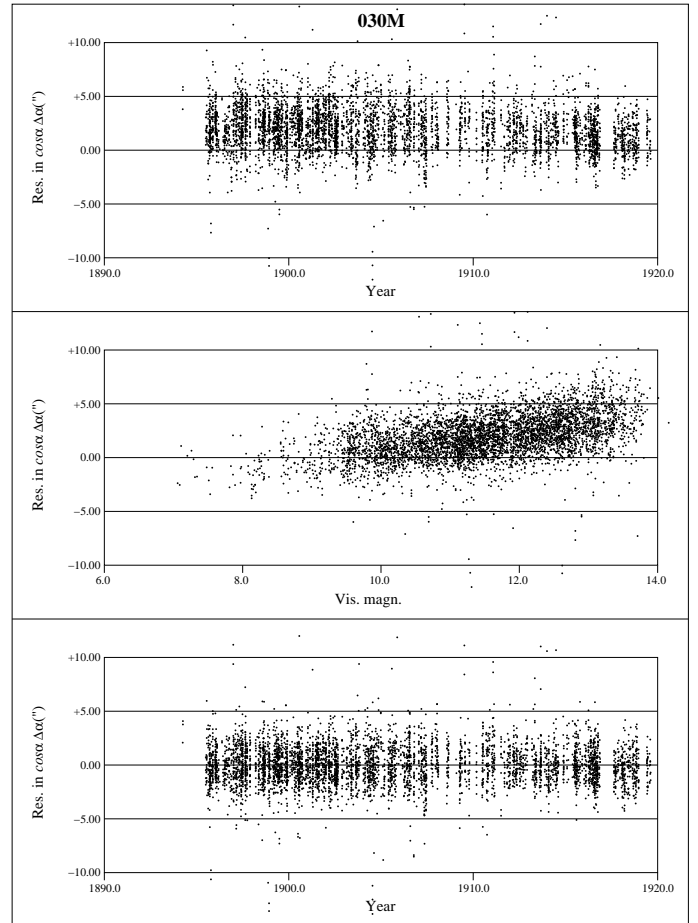


Fig. 1. Magnitude equation in right ascension for the visual micrometer observations made at 030 Arcetri (5147 observations, 1894–1919). Top graph: residuals versus time, uncorrected ($\mu = +1.94''$, $\sigma = 2.07''$). Middle graph: residuals versus visual magnitude of the asteroid; the least-squares regression line is $R = -6.92 + (0.77 \pm 0.01)M_V$. Bottom graph: residuals versus time after magnitude correction ($\mu = 0.00''$, $\sigma = 1.86''$).

- With outliers removed and after correcting for magnitude equation, the next step is then to compute for each time bin the values of the mean μ (the bias), the standard deviation σ and the standard deviation of the mean

$$\sigma_\mu = \frac{\sigma}{\sqrt{N}}$$

where N is the number of remaining residuals in the bin. A bias is accepted as statistically significant when it exceeds 2 times its standard deviation ($|\mu|/\sigma_\mu > 2$). For the observations of code 248 Hipparcos the bias was set equal to zero, since they are by definition on the ICRS system.

The results are applied in *PI* as follows. Observations are considered as outliers, i.e. are given zero weight, if their residuals deviate more than 3σ from the mean. The weight of an observation is calculated as $w = 0.1/\sigma^2$ (unit weight thus corresponds to $\sigma = \sqrt{0.1} \approx 0.32''$). If there are n observations of the same asteroid by the same observatory (except for code 248 Hipparcos) on one night, then their weight is further divided by \sqrt{n} , so that the total weight of the n observations is not wn but $w\sqrt{n}$. This procedure is a simple way to avoid overweighting them with respect to older measurements. The reason is that their residuals

Table 1. Minor planets for which *no* acceptable mass could be calculated. Those above the horizontal line gave negative masses and were removed, those below gave unacceptable results (see text) but were retained as perturbers in the force model with their estimated mass. The table gives: minor planet number and name, taxonomic class, adopted diameter and estimated mass.

Minor planet	Tax. class	Diam. (km)	Est. mass ($10^{-10} M_{\odot}$)	Minor planet	Tax. class	Diam. (km)	Est. mass ($10^{-10} M_{\odot}$)	Minor planet	Tax. class	Diam. (km)	Est. mass ($10^{-10} M_{\odot}$)
12 Victoria	S	127	0.012	225 Henrietta	F	109	0.007	602 Marianna	C	127	0.010
30 Urania	S	98	0.006	233 Asterope	T	100	0.005	675 Ludmilla	S	109	0.008
36 Atalante	C	103	0.005	247 Eukrate	C	134	0.011	712 Boliviana	C	120	0.008
47 Aglaja	C	127	0.010	266 Aline	C	109	0.006	713 Luscinia	C	103	0.005
57 Mnemosyne	S	123	0.011	276 Adelheid	C	103	0.005	747 Winchester	P	184	0.033
78 Diana	C	121	0.008	303 Josephina	F	106	0.006	748 Simeisa	P	103	0.006
86 Semele	C	115	0.007	361 Bononia	D	142	0.015	769 Tatjana	X	103	0.006
97 Klotho	M	83	0.006	386 Siegena	C	169	0.023	776 Berbericia	C	151	0.016
98 Ianthé	C	111	0.007	420 Bertholda	P	144	0.016	788 Hohensteina	C	118	0.008
111 Ate	C	129	0.010	455 Bruchsalia	C	112	0.007	814 Tauris	C	110	0.006
134 Sophrosyne	C	112	0.007	471 Papagena	S	134	0.014	895 Helio	F	119	0.009
137 Meliboea	C	144	0.014	489 Comacina	C	139	0.013	909 Ulla	C	116	0.008
140 Siwa	P	110	0.007	498 Tokia	M	82	0.006	980 Anacostia	S	96	0.005
153 Hilda	P	171	0.026	505 Cava	F	105	0.006	1266 Tone	M	83	0.006
156 Xanthippe	C	111	0.006	506 Marion	C	106	0.006	1268 Libya	M	94	0.009
162 Laurentia	S	104	0.007	508 Princetonia	C	120	0.008	1390 Abastumani	C	108	0.006
176 Iduna	G	122	0.010	514 Armida	C	106	0.006	1467 Mashona	C	104	0.005
190 Ismene	P	127	0.011	521 Brixia	C	111	0.006	1754 Cunningham	M	80	0.006
201 Penelope	M	88	0.008	536 Merapi	C	165	0.021				
209 Dido	C	124	0.009	545 Messalina	C	113	0.007				
21 Lutetia	M	105	0.013	181 Eucharis	S	128	0.012	674 Rachele	S	96	0.005
42 Isis	S	100	0.006	227 Philosophia	F	105	0.006	733 Mocia	C	109	0.006
54 Alexandra	C	142	0.014	229 Adelinda	B	107	0.006	739 Mandeville	C	104	0.005
63 Ausonia	S	103	0.006	241 Germania	C	192	0.034	751 Faïna	C	106	0.006
91 Aegina	C	103	0.005	283 Emma	C	135	0.012	762 Pulcova	F	137	0.014
93 Minerva	C	152	0.017	322 Phaeo	M	77	0.005	780 Armenia	F	102	0.006
105 Artemis	C	105	0.005	344 Desiderata	C	126	0.009	849 Ara	M	84	0.007
135 Hertha	M	77	0.005	348 May	M	83	0.006	1093 Freda	C	117	0.007
141 Lumen	C	137	0.012	387 Aquitania	S	101	0.006	1177 Gonnessia	M	92	0.009
146 Lucina	C	132	0.011	410 Chloris	C	119	0.008	1212 Francette	M	82	0.006
147 Protogeneia	C	133	0.011	469 Argentina	X	122	0.009	1269 Rollandia	D	105	0.006
165 Loreley	C	155	0.018	490 Veritas	C	116	0.007	1911 Schubart	M	80	0.006
168 Sibylla	C	144	0.014	570 Kythera	S	110	0.008				
175 Andromache	C	115	0.007	595 Polyxena	C	108	0.006				

are strongly correlated because the observations were very probably made using the same reference stars.

5. Results

The computations with the compact storage formulation (Eq. 3, with $N = 300\,000$ and $M = 105$) had indicated that the results were sensitive to the weighting of the observations, and in particular to the recomputation of the statistics. Therefore, and in order to avoid a possible initial divergence of the iterations, the first three iterations were done without changing the weights or recomputing the perturber positions, and the recalculation of the observation statistics ($P4$) was only set in after the 6th iteration (and then every 3 iterations).

After the main calculation was finished, a number of test computations were performed to assess the sensitivity of the solution to the starting values of the masses.

5.1. Main calculation

At the start of the computations the masses of the 230 perturbing asteroids were set to their estimated values. After five iterations

it turned out that 58 of them persistently yielded negative mass values in the first solution of the (unmodified) normal equations; they are listed in Table 1 (upper part). It was decided to remove them and to restart the computations with the 172 remaining perturbing asteroids, partly to reduce computing time².

Due to the improvement of the N orbits and the reweighting of the observations at each iteration, the mean error of one observation decreased to $0.23''$ and the total number of equations of condition used in the solution increased from 164.28 to 168.51 million. Convergence for most masses was reached after 14 iterations, i.e. after that their values started to oscillate by a few units of the 4th decimal. The mass of Pallas however showed a general decline and later started fluctuating by one or two units of the 3rd decimal. The calculations were finally stopped after the 29th iteration; the total number of linear equations used in the last iteration was 168 506 583.

For 40 of the 172 perturbing asteroids no acceptable mass could be derived according to the significance and density criteria given above. They are listed in Table 1 (lower part) together

² For the calculations with 230 asteroid masses, the total reported CPU time of the programme *P1* was about 96.5 hours per iteration, slightly over 59 hours with 172 perturbers.

Table 2. Twenty closest approaches between test and perturbing asteroids.

Perturbing asteroid	Test asteroid	Min. dist (au)
324 Bamberga	310585 2001 UU ₂₀	0.00006
50 Virginia	79009 4707 P-L	0.00014
20 Massalia	336058 2008 EU ₃₆	0.00015
110 Lydia	66778 1999 TL ₂₂₁	0.00016
324 Bamberga	121416 1999 TG ₁₄₆	0.00019
96 Aegle	194481 2001 WA ₃₇	0.00019
49 Pales	145746 1995 UR ₁₈	0.00023
106 Dione	233456 2006 JS ₃₈	0.00026
426 Hippos	308965 2006 TG ₆₇	0.00033
203 Pompeja	204819 2007 PU ₁	0.00034
40 Harmonia	272519 2005 UA ₂₅₆	0.00035
187 Lambertia	206779 2004 CR ₉₇	0.00035
56 Melete	22960 1999 UE ₄	0.00035
38 Leda	47863 2000 EC ₁₈₀	0.00035
15 Eunomia	50278 2000 CZ ₁₂	0.00038
128 Nemesis	272917 2006 BR ₁₈₀	0.00039
211 Isolda	257257 2009 FB ₄₃	0.00042
11 Parthenope	334240 2001 TW ₉₆	0.00045
110 Lydia	70797 1999 VF ₅₄	0.00046
74 Galatea	63721 2001 QH ₂₃₄	0.00047

Table 3. Ten largest correlations between the masses of perturbers and the semi-major axes of the test asteroids.

Perturbing asteroid	Test asteroid	Corr. coeff.
2 Pallas	58243 1993 NG ₁	+0.99
2 Pallas	133978 2004 TK ₂₄₂	+0.97
15 Eunomia	50278 2000 CZ ₁₂	-0.96
49 Pales	145746 1995 UR ₁₈	+0.96
88 Thisbe	70336 1999 RO ₁₆₉	+0.96
196 Philomela	104441 2000 GQ	-0.95
419 Aurelia	118346 1999 CA ₄₉	-0.95
9 Metis	36653 2000 QF ₂₀₀	-0.94
106 Dione	233456 2006 JS ₃₈	-0.93
423 Diotima	72812 2001 GB ₈	-0.93

Table 4. Ten largest mutual correlations between the perturber masses.

Perturber 1	Perturber 2	Corr. coeff.
17 Thetis	145 Adeona	+0.56
4 Vesta	11 Parthenope	-0.45
17 Thetis	275 Sapiientia	+0.39
11 Parthenope	40 Harmonia	-0.24
88 Thisbe	89 Julia	+0.22
49 Pales	175 Andromache	-0.18
5 Astraea	356 Liguria	-0.17
2 Pallas	203 Pompeja	-0.17
76 Freia	203 Pompeja	-0.16
139 Juewa	212 Medea	+0.16

with their taxonomic class, estimated diameter and mass used in the dynamical model. Table 2 gives the 20 closest approaches between test and perturbing asteroids. Table 5 gives the 132 asteroids with acceptable astrometric masses, together with their adopted (equivalent) diameter, taxonomic class and bulk density derived from these quantities. The results for 50 of them appear to be new; they are indicated by an asterisk in the table.

As is to be expected, there are high correlations between the perturber masses and the semi-major axes of some of the test asteroids. The 10 largest values are given in Table 3; all of them involve high-numbered test asteroids. The mutual correlations between the M masses turn out to be generally low; the 10 largest values are listed in Table 4. All correlations between the 10 *largest* asteroid masses are ≤ 0.05 .

5.2. Influence of initial assumed masses

After concluding the main calculation, the perturber masses were set to zero and the iterations were resumed *without* reweighting the observations, i.e. with the iteration sequence $P1 + P2 + P3$. After three iterations, the masses were back to the results of the main calculation to within four decimal places. Next, the masses were set to twice their assumed values and the calculations were continued. Again, three iterations sufficed. This indicates that the least-squares algorithm is rather insensitive to the starting mass values.

Then, the *main* calculations as described above were repeated twice, first with all perturber masses initially set to zero and then set to twice their estimated values, but this time with reweighting applied. The statistical analysis obtained at the end of the previous set of iterations was used at the beginning of the new test series. Convergence was deemed to be reached in both cases after 20 iterations; the number of linear equations in the last iteration steps was less than in the main calculation: -20 222 (-0.012%) and -30 237 (-0.018%) respectively. In Table 5 the mass, standard deviation and significance are from the main iteration. The mass differences between the last two test iterations and the main one are also included (column Δm_0 for the zero and column Δm_2 for the double starting mass values).

6. Discussion

The masses given in Table 5 are presented "as such", i.e. as the outcome of the least-squares adjustment described above. The bulk density is a quantity derived from the mass and the assumed diameter and its uncertainty is thus affected by the uncertainties of both the mass and the diameter. No physical interpretation of the densities will be attempted here. Some results conform to what is to be expected for their taxonomic class, while others are higher or lower. One should bear in mind that the taxonomic class is derived from the spectral properties of the surface of an asteroid and as such does not say much about its internal composition or porosity.

The mass and size of 4 Vesta is accurately known from the orbit of the Dawn spacecraft. Russell et al. (2012) give a value of $2.59076 \pm 0.00001 \times 10^{20}$ kg corresponding to $1.30260 \pm 0.00001 \times 10^{-10} M_\odot$, an equivalent diameter of 523.15 km and a density of 3.456 g/cm^3 . The result from this study, to 4 decimal places, is 1.2995 ± 0.0015 yielding a density of 3.448 g/cm^3 , in good agreement with the Dawn result.

It would be a constructive achievement to develop an objective criterion to exclude test asteroids that do not significantly contribute to the solution. The correlations between the masses and the semi-major axes come to mind as an obvious choice. Using the sum of the absolute values of the M correlation coefficients as an indicator for the usefulness of a test asteroid, test calculations were made with $N = 10,000$ and $M = 10$ (the 10 largest masses). With a limit for the sum of 0.01, about 15% were excluded and the average difference in the 10 masses was 0.0003; this became 48% and 0.0012 with 0.02 as the limit. It

seems worthwhile to test this criterion on the full set of test asteroids.

In general, the two test calculations clearly show that the most influential factor is the treatment of the observations (weighting and outlier rejection). In that respect, it should be remarked that the observations reduced with the USNO A1.0, A 2.0 and B 1.0 star catalogues have not been corrected for the systematic biases as described in Chesley et al. (2010) or in Röser et al. (2010). It should be worthwhile to include these in future work.

The planetary ephemeris uncertainties are negligible in this case, certainly the positional errors. The planetary masses come from spacecraft encounters, and DE421/422 is up-to-date. The best asteroid-produced planetary mass of Jupiter, for example, was inaccurate by about 6 parts in 10^6 ; the spacecraft-produced values are two orders of magnitude more accurate.

The Gaia satellite, launched at the end of 2013, will in the course of its 5 year nominal duration systematically observe $\sim 300\,000$ asteroids brighter than $V = 20$ with an accuracy of 0.3–5 mas. This will enable the determination of the masses of about 150 individual asteroids from close encounters, ~ 100 with an accuracy of $\leq 50\%$ and ~ 50 with an accuracy of $\leq 10\%$ (Mouret et al. 2008). The Gaia star catalogue will enable the reduction of future, and the re-reduction of old observations with a very high level of accuracy (about 10 mas, see Desmars et al. 2013), thus considerably increasing the accuracy of simultaneous asteroid mass determinations.

Finally, Table 6 gives an overview of mass results from recent determinations for those minor planets given in Table 5, and published in roughly the last ten years. For a more complete list including older determinations, the reader is referred to Zielenbach (2011). Besides dynamical determinations using asteroids as test objects, there are also determinations from natural satellites and from planetary ephemerides. Note that these use a completely different type of observations.

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Table 5. Masses of 132 asteroids obtained from the simultaneous solution. The table gives: minor planet number and name (* = new mass determination), mass, standard deviation and significance, mass difference when starting from masses equal to zero (Δm_0) and from masses twice (Δm_2) the assumed values, adopted equivalent diameter, taxonomic class and resulting bulk density. Standard deviations printed as "0.000" indicate values < 0.0005 ; their exact value can be inferred from the values of the mass and its significance.

Minor planet	Mass ($10^{-10} M_\odot$)	St. dev.	Signi- ficance	Δm_0 ($10^{-13} M_\odot$)	Δm_2	Diam. (km)	Taxon. class	Dens. (g/cm ³)
1 Ceres	4.748	0.003	1561.4	+1.1	+0.5	953	G	2.09
2 Pallas	1.007	0.011	88.1	-2.9	-2.7	531	B	2.56
3 Juno	0.126	0.004	31.6	-1.5	-0.5	258	S	2.81
4 Vesta	1.300	0.002	847.2	-0.2	-0.1	523	V	3.45
5 Astraea	0.033	0.003	12.4	+0.3	+0.2	127	S	6.10
6 Hebe	0.045	0.003	13.4	-1.2	-0.7	186	S	2.64
7 Iris	0.070	0.002	34.9	-0.5	-0.3	213	S	2.78
8 Flora	0.029	0.002	17.0	-2.1	-2.5	161	S	2.65
9 Metis	0.025	0.002	15.9	+0.4	+0.3	174	S	1.84
10 Hygiea	0.410	0.004	115.8	-0.3	-0.3	431	C	1.95
11 Parthenope	0.025	0.000	69.5	0.0	+0.1	153	S	2.67
13 Egeria	0.047	0.004	12.0	+0.2	+0.1	208	G	1.97
14 Irene	0.028	0.003	9.6	-1.1	-1.1	152	S	2.97
15 Eunomia	0.146	0.002	64.3	-0.4	-0.3	268	S	2.88
16 Psyche	0.117	0.002	56.6	-0.7	-0.5	186	M	6.89
17 Thetis	0.004	0.000	12.5	-0.1	-0.1	98	S	1.51
18 Melpomene	0.027	0.003	10.2	+0.9	+0.6	150	S	3.05
19 Fortuna	0.045	0.001	35.3	-0.3	-1.4	208	G	1.89
20 Massalia	0.024	0.001	19.7	-0.3	0.0	145	S	3.01
22 Kalliope	0.024	0.004	6.4	+0.3	-0.1	175	M	1.73
23 Thalia	0.023	0.003	7.5	+0.8	+0.9	108	S	7.06
24 Themis	0.051	0.003	16.3	-0.7	-0.3	198	C	2.50
27 Euterpe	0.010	0.002	5.0	+0.2	+0.3	118	S	2.23
28 Bellona	0.008	0.003	2.9	-0.5	0.0	121	S	1.66
29 Amphitrite	0.067	0.002	44.0	+0.2	+0.2	212	S	2.67
31 Euphrosyne	0.087	0.007	12.7	-0.1	-0.9	256	C	1.96
34 Circe*	0.021	0.002	9.0	-0.7	-0.8	113	C	5.40
37 Fides*	0.012	0.002	6.5	-0.5	-0.4	108	S	3.49
38 Leda*	0.016	0.003	6.2	+1.1	+1.4	116	C	3.79
39 Laetitia	0.022	0.003	7.5	-0.4	-0.1	163	S	1.93
40 Harmonia*	0.020	0.002	10.5	+0.2	+0.5	120	S	4.35
41 Daphne	0.047	0.007	7.2	-0.9	-0.6	177	C	3.25
45 Eugenia	0.032	0.002	17.0	-1.2	-1.3	193	F	1.69
46 Hestia	0.005	0.002	2.8	0.0	0.0	124	P	0.95
48 Doris	0.039	0.003	11.8	-0.6	-0.6	222	C	1.35
49 Pales	0.027	0.003	10.5	-0.4	-0.7	150	C	3.09
50 Virginia*	0.003	0.000	6.5	0.0	+0.1	100	X	1.02
51 Nemausa	0.014	0.002	8.1	-0.1	-0.5	145	C	1.81
52 Europa	0.135	0.003	43.5	-0.5	-0.7	315	C	1.82
53 Kalypso*	0.008	0.002	3.7	0.0	+0.2	115	X	2.12
56 Melete*	0.019	0.003	7.1	-0.9	-1.0	129	P	3.33
59 Elpis	0.017	0.003	6.4	-0.4	-0.5	166	C	1.43
65 Cybele	0.089	0.004	24.0	-0.4	-0.9	273	P	1.66
68 Leto	0.021	0.003	7.9	+0.9	+0.6	129	S	3.72
69 Hesperia	0.047	0.004	11.1	-0.4	-1.0	138	M	6.82
70 Panopaea*	0.017	0.002	8.3	+0.1	+0.1	139	C	2.38
74 Galatea*	0.005	0.002	2.3	+0.4	+0.3	119	C	1.24
76 Freia*	0.038	0.004	9.0	+0.4	+0.1	159	P	3.64
81 Terpsichore*	0.016	0.001	11.5	0.0	-0.1	122	C	3.31
85 Io	0.012	0.003	3.5	-0.3	0.0	164	F	1.02
87 Sylvia	0.108	0.010	11.0	-0.3	-3.2	286	P	1.76
88 Thisbe	0.051	0.002	20.5	+0.4	+0.4	225	C	1.69
89 Julia	0.043	0.003	13.1	-0.8	-1.0	148	S	5.05
92 Undina*	0.037	0.005	7.0	-0.7	+0.2	126	X	6.93
94 Aurora	0.009	0.003	3.2	+0.1	0.0	178	C	0.59
95 Arethusa*	0.021	0.004	5.6	+0.3	+0.2	145	C	2.61
96 Aegle	0.057	0.003	17.9	+0.2	+0.6	170	T	4.38

Table 5. continued.

Minor planet	Mass ($10^{-10} M_{\odot}$)	St. dev.	Signi- ficance	Δm_0 ($10^{-13} M_{\odot}$)	Δm_2	Diam. (km)	Taxon. class	Dens. (g/cm ³)
104 Klymene*	0.009	0.002	3.7	0.0	-0.1	126	C	1.66
106 Dione	0.018	0.003	5.5	+0.3	+0.3	136	G	2.77
107 Camilla	0.056	0.009	6.4	-0.2	-1.4	259	C	1.22
110 Lydia*	0.007	0.000	22.4	0.0	0.0	89	M	3.89
114 Kassandra*	0.005	0.002	2.1	+0.4	-0.1	100	T	1.81
117 Lomia	0.017	0.004	4.0	-0.7	-0.9	173	C	1.24
120 Lachesis*	0.032	0.004	7.7	-0.9	-0.5	163	C	2.83
121 Hermione	0.016	0.002	7.2	-0.4	-0.5	187	C	0.96
128 Nemesis	0.042	0.003	16.3	-0.3	-0.2	188	C	2.37
129 Antigone	0.032	0.004	9.1	-0.2	-0.3	130	M	5.67
130 Elektra	0.070	0.008	8.9	-0.7	-1.5	215	G	2.67
139 Juewa	0.030	0.003	9.0	+0.4	+0.3	164	C	2.57
144 Vibia	0.017	0.002	7.6	0.0	+0.1	152	C	1.83
145 Adeona	0.014	0.003	4.6	+0.6	+0.9	151	C	1.55
150 Nuwa	0.017	0.003	6.3	-0.3	-0.5	137	C	2.51
154 Bertha	0.081	0.008	9.8	+1.6	+0.8	189	C	4.57
159 Aemilia*	0.021	0.003	7.4	-0.7	-0.5	127	C	3.94
164 Eva*	0.011	0.006	1.6	-0.6	-0.1	109	C	3.10
171 Ophelia*	0.006	0.003	2.2	+0.2	+0.1	104	C	2.13
173 Ino	0.016	0.003	5.8	-0.5	-2.2	154	C	1.68
185 Eunike*	0.021	0.006	3.2	+0.9	+1.1	156	C	2.08
187 Lambertia	0.011	0.004	3.1	+0.6	+0.2	133	C	1.72
194 Prokne	0.023	0.003	6.9	+0.3	+0.4	169	C	1.83
196 Philomela	0.027	0.004	6.3	0.0	-0.5	158	S	2.64
200 Dynamene*	0.020	0.003	7.9	-0.6	-0.8	131	C	3.42
202 Chryseis*	0.015	0.002	7.4	-0.2	-0.2	98	S	5.98
203 Pompeja*	0.006	0.002	4.1	-0.1	-0.2	111	D	1.75
206 Hersilia*	0.018	0.002	11.9	-0.4	-0.4	105	C	6.05
211 Isolda	0.015	0.003	6.0	+0.1	+0.3	143	C	1.94
212 Medea	0.011	0.001	10.2	+0.1	+0.1	136	D	1.65
216 Kleopatra	0.026	0.004	6.4	+0.9	+0.3	138	M	3.83
221 Eos*	0.012	0.003	3.7	+0.6	+0.5	96	S	5.40
230 Athamantis*	0.012	0.002	5.0	+0.2	-0.4	109	S	3.57
238 Hypatia	0.015	0.003	4.6	-0.1	0.0	145	C	1.83
250 Bettina*	0.013	0.005	2.4	-0.2	+0.2	121	M	2.69
259 Aletheia	0.065	0.005	13.8	-0.4	-1.0	207	C	2.79
268 Adorea	0.016	0.002	6.8	+0.1	+0.1	141	F	2.14
275 Sapientia*	0.013	0.003	5.0	+0.5	-0.1	103	C	4.50
308 Polyxo*	0.024	0.002	14.5	-0.8	-0.7	144	T	2.99
324 Bamberga	0.044	0.002	18.7	-0.8	-0.8	212	C	1.77
328 Gudrun	0.029	0.007	4.3	-1.4	-1.3	116	S	6.94
334 Chicago*	0.094	0.018	5.1	+1.2	-0.8	173	C	6.92
349 Dembowska	0.014	0.003	4.8	+0.3	0.0	143	R	1.82
354 Eleonora	0.089	0.006	15.4	+0.2	+1.0	165	S	7.53
356 Liguria*	0.014	0.002	8.5	+0.4	+0.4	131	C	2.43
360 Carlova*	0.034	0.003	10.6	0.0	-1.1	133	C	5.60
372 Palma*	0.032	0.005	6.1	+0.3	+0.3	191	B	1.74
375 Ursula	0.059	0.006	10.2	-0.1	-1.0	204	C	2.65
381 Myrrha*	0.035	0.005	7.0	+0.2	0.0	129	C	6.18
388 Charybdis*	0.027	0.003	8.0	-0.2	-0.4	124	C	5.30
405 Thia*	0.011	0.005	2.0	+0.8	+1.2	125	C	2.09
409 Aspasia	0.027	0.003	9.5	+0.9	+0.9	163	C	2.41
419 Aurelia*	0.022	0.002	11.6	-0.3	0.0	129	F	3.87
423 Diotima	0.048	0.004	11.6	-0.1	+0.3	170	C	3.73
426 Hippo*	0.011	0.003	3.9	+0.4	-0.1	116	F	2.56
444 Gyptis*	0.007	0.003	2.3	+1.7	+1.8	163	C	0.62
445 Edna*	0.021	0.006	3.3	-0.9	-0.8	106	C	6.70
451 Patientia	0.068	0.005	14.6	+1.2	+1.3	225	C	2.27
476 Hedwig*	0.007	0.002	3.0	0.0	+0.1	117	P	1.65
488 Kreusa	0.037	0.005	7.9	-0.4	-0.4	150	C	4.14

Table 5. continued.

Minor planet	Mass ($10^{-10}M_{\odot}$)	St. dev.	Signi- ficance	Δm_0 ($10^{-13}M_{\odot}$)	Δm_2	Diam. (km)	Taxon. class	Dens. (g/cm ³)
511 Davida	0.151	0.005	30.9	+0.0	+0.0	289	C	2.37
532 Herculina	0.104	0.004	26.5	+0.8	+0.7	222	S	3.60
554 Peraga*	0.004	0.001	3.0	-0.1	-0.1	105	F	1.30
566 Stereoscopia*	0.031	0.005	6.1	-1.0	-1.0	134	C	4.97
596 Scheila*	0.027	0.006	4.5	-0.6	-0.3	114	P	6.87
618 Elfriede*	0.016	0.005	3.1	-0.2	-0.1	131	C	2.62
654 Zelinda*	0.034	0.005	6.4	+0.2	+0.1	120	C	7.38
690 Wratislavia	0.011	0.005	2.2	-0.6	0.0	135	C	1.76
702 Alauda	0.066	0.010	6.7	+0.0	0.0	207	C	2.82
704 Interamnia	0.137	0.006	24.0	-0.8	0.0	319	F	1.60
705 Erminia*	0.039	0.006	6.4	-3.6	-2.0	131	C	6.63
772 Tanete*	0.058	0.009	6.8	+0.5	-0.4	143	C	7.63
790 Pretoria	0.049	0.009	5.1	+2.7	+3.2	170	P	3.74
804 Hispania	0.014	0.003	4.9	0.0	-0.3	147	P	1.69
1428 Mombasa*	0.019	0.004	4.4	-0.3	-0.5	127	S	3.54

Table 6. Mass determinations of minor planets from recent dynamical results and latest planetary ephemerides. The table gives: minor planet number and name, mass and standard deviation and reference (^p = planetary ephemerides, ^s = binary or natural satellites).

Minor planet	Mass ($10^{-10}M_{\odot}$)	St. Dev.	Ref.
1 Ceres	4.759	0.023	1
	4.70	0.04	2
	4.736	0.026	3
	4.75	0.03	4
	4.685		5 ^p
	4.712	0.006	6
	4.757	0.007	8
	4.679	0.033	9
	4.735	0.013	10
	4.731	0.029	37 ^p
	4.673	0.019	7 ^p
	4.748	0.003	99
2 Pallas	1.21	0.26	2
	1.17	0.03	11
	1.06	0.13	4
	1.010		5 ^p
	1.027	0.007	6
	1.010	0.065	8
	1.034	0.069	10
	1.017	0.021	37 ^p
	1.127	0.016	7 ^p
	1.007	0.011	99
3 Juno	0.209	0.035	14
	0.144	0.023	8
	0.156	0.016	10
	0.135	0.008	37 ^p
	0.118	0.006	7 ^p
4 Vesta	0.126	0.004	99
	1.36	0.05	2
	1.306	0.016	15
	1.282	0.018	16
	1.34	0.01	4
	1.328		5 ^p
	1.344	0.003	6
	1.30	0.00	2
	1.300	0.005	8
	1.303	0.007	10
	1.303	0.000	34 ^s
	1.305	0.008	37 ^p
	1.303	0.009	7 ^p
	1.300	0.002	99
5 Astraea	0.043	0.011	10
	0.033	0.003	99
6 Hebe	0.069	0.022	17
	0.069	0.009	14
	0.076	0.014	4
	0.064	0.007	8
	0.077	0.012	10
	0.042	0.010	37 ^p
	0.071	0.008	7 ^p
7 Iris	0.045	0.003	99
	0.141	0.014	14
	0.24	0.03	19
	0.068	0.019	4
	0.081	0.005	8
	0.084	0.008	10
	0.075	0.008	37 ^p

Table 6. continued.

Minor planet	Mass ($10^{-10}M_{\odot}$)	St. Dev.	Ref.
	0.063	0.006	7 ^p
	0.070	0.002	99
8 Flora	0.043	0.005	18
	0.033	0.004	8
	0.037	0.007	10
	0.022	0.004	37 ^p
	0.034	0.004	7 ^p
	0.029	0.002	99
9 Metis	0.103	0.024	4
	0.057	0.014	18
	0.012	0.010	7
	0.057	0.011	8
	0.045	0.007	10
	0.035	0.007	37 ^p
	0.042	0.007	7 ^p
	0.025	0.002	99
10 Hygieia	0.557	0.070	17
	0.501	0.041	14
	0.25	0.04	19
	0.454	0.015	4
	0.436	0.007	8
	0.413	0.015	10
	0.528	0.042	37 ^p
	0.440	0.032	7 ^p
	0.410	0.004	99
11 Parthenope	0.026	0.001	15
	0.032	0.001	4
	0.029	0.001	10
	0.02	0.00	9
13 Egeria	0.082	0.016	18
	0.080	0.022	8
	0.031	0.016	10
	0.062	0.017	37 ^p
	0.047	0.008	7 ^p
	0.047	0.004	99
14 Irene	0.041	0.007	18
	0.035	0.008	8
	0.010	0.010	10
	0.033	0.007	37 ^p
	0.028	0.003	99
15 Eunomia	0.126	0.030	17
	0.106	0.016	14
	0.164	0.006	21
	0.08	0.03	19
	0.168	0.008	4
	0.162	0.005	22
	0.160	0.002	8
	0.162	0.004	10
	0.131	0.014	37 ^p
	0.158	0.010	7 ^p
	0.146	0.002	99
16 Psyche	0.087	0.026	23
	1.27	0.18	24
	0.338	0.028	25
	0.134	0.022	14
	0.40	0.14	19
	0.129	0.017	4
	0.114	0.004	8
	0.123	0.008	10
	0.089	0.021	67 ^p

Table 6. continued.

Minor planet	Mass ($10^{-10}M_{\odot}$)	St. Dev.	Ref.
	0.126	0.022	7 ^p
	0.117	0.002	99
17 Thetis	0.006	0.001	4
	0.007	0.000	8
	0.037	0.009	7 ^p
	0.004	0.000	99
18 Melpomene	0.015	0.005	18
	0.025	0.010	10
	0.020	0.005	37 ^p
	0.027	0.003	99
19 Fortuna	0.054	0.008	4
	0.064	0.003	18
	0.042	0.004	8
	0.051	0.005	10
	0.039	0.005	37 ^p
	0.049	0.005	7 ^p
	0.045	0.001	99
20 Massalia	0.024	0.004	26
	0.029	0.004	18
	0.017	0.004	8
	0.030	0.006	10
	0.029	0.011	7 ^p
	0.024	0.001	99
22 Kalliope	0.037	0.002	27
	0.085	0.028	14
	0.041	0.001	29 ^s
	0.041	0.001	28 ^s
	0.066	0.019	10
	0.084	0.007	7 ^p
	0.024	0.004	99
23 Thalia	0.010		5 ^p
	0.015	0.003	7 ^p
	0.023	0.003	99
24 Themis	0.057	0.022	4
	0.028	0.019	7 ^p
	0.031	0.009	10
	0.051	0.003	99
27 Euterpe	0.011	0.007	10
	0.010	0.002	99
28 Bellona	0.07	0.02	19
	0.008	0.003	99
29 Amphitrite	0.077	0.012	14
	0.100	0.035	4
	0.076	0.003	8
	0.056	0.008	10
	0.052	0.012	37 ^p
	0.072	0.010	7 ^p
	0.067	0.002	99
31 Euphrosyne	0.094	0.052	14
	0.313	0.059	18
	0.292	0.100	8
	0.135	0.046	10
	0.110	0.037	37 ^p
	0.132	0.020	7 ^p
	0.087	0.007	99
39 Laetitia	0.088	0.005	7 ^p
	0.028	0.007	8
	0.022	0.013	10
	0.022	0.003	99
41 Daphne	0.092	0.026	7 ^p

Table 6. continued.

Minor planet	Mass ($10^{-10}M_{\odot}$)	St. Dev.	Ref.
	0.024	0.028	10
	0.039	0.009	37 ^p
	0.047	0.007	99
45 Eugenia	0.030	0.001	30
	0.09	0.03	19
	0.029	0.001	29 ^s
	0.038	0.008	10
	0.032	0.002	99
46 Hestia	0.109	0.068	13
	0.035	0.007	7 ^p
	0.005	0.002	99
48 Doris	0.061	0.030	14
	0.201	0.150	7 ^p
	0.019	0.019	10
	0.039	0.003	99
49 Pales	0.014	0.003	18
	0.027	0.003	99
51 Nemausa	0.017	0.008	10
	0.000	0.000	7 ^p
	0.014	0.002	99
52 Europa	0.261	0.088	17
	0.127	0.025	14
	0.42	0.11	19
	0.098	0.022	4
	0.083	0.008	18
	0.114	0.008	8
	0.140	0.016	10
	0.167	0.041	37 ^p
	0.107	0.026	7 ^p
	0.135	0.003	99
59 Elpis	0.026	0.018	7 ^p
	0.014	0.002	10
	0.017	0.003	99
65 Cybele	0.058	0.015	14
	0.076	0.018	4
	0.089	0.006	18
	0.072	0.043	7 ^p
	0.053	0.010	8
	0.077	0.014	10
	0.089	0.004	99
68 Leto	0.015	0.014	10
	0.021	0.003	99
69 Hesperia	0.033	0.021	10
	0.047	0.005	99
85 Io	0.018	0.013	10
	0.022	0.008	7 ^p
	0.012	0.003	99
87 Sylvia	0.075	0.000	30 ^s
	0.26	0.11	19
	0.059	0.053	10
	0.075	0.001	36 ^s
	0.108	0.010	99
88 Thisbe	0.074	0.013	17
	0.059	0.012	14
	0.057	0.018	4
	0.053	0.009	18
	0.092	0.006	8
	0.038	0.011	10
	0.057	0.017	37 ^p
	0.071	0.014	7 ^p

Table 6. continued.

Minor planet	Mass ($10^{-10}M_{\odot}$)	St. Dev.	Ref.
	0.051	0.002	99
89 Julia	0.042	0.021	7 ^p
	0.043	0.003	99
94 Aurora	0.150	0.034	7 ^p
	0.009	0.003	99
96 Aegle	0.040	0.014	37 ^p
	0.057	0.009	99
106 Dione	0.018	0.013	10
	0.039	0.004	7 ^p
	0.018	0.003	99
107 Camilla	0.056	0.002	29 ^s
	0.088	0.044	10
	0.034	0.015	7 ^p
	0.056	0.009	99
117 Lomia	0.087	0.001	7 ^p
	0.034	0.018	10
	0.017	0.004	99
121 Hermione	0.047	0.008	23
	0.027	0.002	31 ^s
	0.024	0.001	32 ^s
	0.023	0.011	10
	0.016	0.002	99
128 Nemesis	0.042	0.011	10
	0.069	0.016	7 ^p
	0.042	0.003	99
129 Antigone	0.043	0.018	7 ^p
	0.024	0.028	10
	0.032	0.004	99
130 Elektra	0.033	0.002	27 ^s
	0.112	0.080	7 ^p
	0.035	0.032	10
	0.070	0.008	99
139 Juewa	0.030	0.016	10
	0.021	0.009	7 ^p
	0.030	0.003	99
144 Vibia	0.046	0.030	7 ^p
	0.025	0.009	10
	0.017	0.002	99
145 Adeona	0.011	0.015	10
	0.014	0.003	99
150 Nuwa	0.091	0.004	7 ^p
	0.010	0.014	10
	0.017	0.003	99
154 Bertha	0.064	0.048	10
	0.081	0.008	99
173 Ino	0.067	0.015	7 ^p
	0.016	0.003	99
187 Lamberta	0.025	0.011	7 ^p
	0.011	0.003	99
194 Prokne	0.056	0.006	7 ^p
	0.023	0.003	99
196 Philomela	0.017	0.008	10
	0.027	0.004	99
211 Isolda	0.038	0.007	7 ^p
	0.012	0.009	10
	0.015	0.003	99
212 Medea	0.066	0.005	7 ^p
	0.011	0.001	99
216 Kleopatra	0.006	0.005	7 ^p
	0.038	0.011	10

Table 6. continued.

Minor planet	Mass ($10^{-10}M_{\odot}$)	St. Dev.	Ref.
	0.026	0.004	99
238 Hypatia	0.031	0.016	10
	0.015	0.003	99
259 Aletheia	0.037	0.023	10
	0.065	0.005	99
268 Adorea	0.008	0.007	7 ^p
	0.016	0.002	99
324 Bambergga	0.229	0.038	14
	0.054	0.010	10
	0.051	0.004	37 ^p
	0.048	0.004	7 ^p
	0.044	0.002	99
328 Gudrun	0.009	0.026	10
	0.029	0.007	99
349 Dembowska	0.018	0.013	10
	0.014	0.003	99
354 Eleonora	0.060	0.025	10
	0.089	0.006	99
375 Ursula	0.037	0.020	10
	0.059	0.006	99
409 Aspasia	0.017		5 ^p
	0.062	0.016	10
	0.000	0.000	7 ^p
	0.027	0.003	99
423 Diotima	0.034	0.021	10
	0.048	0.004	99
451 Patientia	0.102	0.034	14
	0.056	0.032	10
	0.150	0.036	7 ^p
	0.068	0.005	99
488 Kreusa	0.011	0.034	10
	0.037	0.005	99
511 Davida	0.334	0.028	17
	0.240	0.024	14
	0.298	0.030	4
	0.220	0.010	18
	0.190	0.010	8
	0.131	0.030	10
	0.148	0.042	37 ^p
	0.183	0.029	7 ^p
	0.151	0.005	99
532 Herculina	0.168	0.028	14
	0.088	0.022	10
	0.065	0.014	37 ^p
	0.116	0.001	
	0.104	0.004	99
690 Wratislavia	0.064	0.001	7 ^p
	0.011	0.005	99
702 Alauda	0.031	0.002	35 ^s
	0.069	0.056	7 ^p
	0.066	0.010	99
704 Interamnia	0.352	0.093	17
	0.081	0.042	14
	0.57	0.16	19
	0.358	0.042	4
	0.186	0.011	18
	0.157	0.026	10
	0.198	0.035	37 ^p
	0.192	0.024	7 ^p
	0.137	0.006	99

Table 6. continued.

Minor planet	Mass ($10^{-10} M_{\odot}$)	St. Dev.	Ref.
790 Pretoria	0.022	0.024	10
	0.049	0.009	99
804 Hispania	0.047	0.019	4
	0.020	0.004	18
	0.018	0.004	8
	0.025	0.018	7 ^p
	0.028	0.014	10
	0.014	0.004	99

References. (1) Viateau & Rapaport (1998); (2) Michalak (2000); (3) Kovačević & Kuzmanoski (2007); (4) Baer & Chesley (2008); (5) Folkner et al. (2008); (6) Pitjeva (2009); (7) Fienga et al. (2013); (8) Baer et al. (2011); (9) Konopliv et al. (2011); (10) Zielenbach (2011); (11) Goffin (2001); (13) Bang & Bec-Borsenberger (1997); (14) Kochetova (2004); (15) Viateau & Rapaport (2001); (16) Kovačević (2005); (17) Michalak (2001); (18) Baer et al. (2008); (19) Ivantsov (2008); (20) Kuzmanoski et al. (2010); (21) Vitagliano & Stoss (2006); (22) Zielenbach (2010); (23) Viateau (2000); (24) Krasinsky et al. (2001); (25) Kuzmanoski & Kovačević (2002); (26) Bange (1998); (27) Marchis et al. (2008a); (28) Descamps et al. (2008); (29) Marchis et al. (2008b); (30) Marchis et al. (2005a); (31) Marchis et al. (2005b); (32) Descamps et al. (2009); (33) Merline et al. (2002); (34) Russell et al. (2012); (35) Rojo & Margot (2010); (36) Fang et al. (2012); (37) Kuchynka & Folkner (2013); (99) This paper.